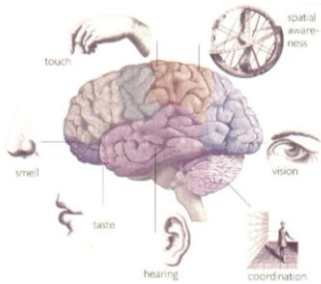
THE NEUROPSYCHOLOGY OF MATHEMATICS



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Presentation Of Goals

(1) Discuss the international trends in math, and reasons why the United States lags behind some industrialized nations in math and science.

(2) Introduce a *brain-based* educational model of math by identifying three basic neural codes which format numbers in the brain.

(3) Explore the role of various cognitive constructs including working memory, visual-spatial functioning, and executive functioning, with respect to math problem solving ability.

(4) Introduce the 90 minute assessment model of mathematics and interventions.

2007 TIMSS DATA

- The Trends in International Mathematics and Science Study (TIMSS) measures mathematical content domains including *number-sense, geometrical shapes, measurement, algebraic skills*, and *data displays* in both ***4th*** and ***8th*** grades. There were ***36*** countries participating in grade 4, and ***47*** countries participating in grade 8. Key findings include:
  - **In 2007, U.S. 4th graders (529) and 8th graders (508) were higher than the TIMSS average of 500.**
  - **Compared to 1995, the average math score for U.S. 4th graders (511) and 8th graders (492) was higher.**
  - **In 2007, 10% of 4th graders and 6% of 8th graders scored at or above the advanced international benchmarks for mathematics.**
  - **At grade 8, there were no measureable differences between U.S. males and females, though at grade 4, there was a 6 point male advantage.**

2007 TIMSS DATA 4th Grade

Country	Average Score
International Average	*500*
Hong Kong	607
Singapore	599
Chinese Taipei	576
Japan	568
Kazakhstan	549
Russian Federation	544
England	541
Latvia	537
Netherlands	535
Lithuania	530
UNITED STATES	529
Germany	525
Denmark	523
Australia	516
Hungary	510
Italy	507
Austria	505
Sweden	503
Slovenia	502
Armenia	500
Slovak Republic	496
Scotland	494
New Zealand	492

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2007 TIMSS DATA 8th Grade

Country	Average Score
International Average	*500*
Chinese Tapai	598
Korea	597
Singapore	593
Hong Kong	572
Japan	570
Hungary	517
England	513
Russian Federation	512
*UNITED STATES	508
Lithuania	506
Czech Republic	504
Slovenia	501
Armenia	499
Australia	496
Sweden	491
Malta	488
Scotland	487
Serbia	486
Italy	480
Malaysia	474
Norway	469
Cyprus	465
Bulgaria	464
Israel	463
Ukraine	462

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PISA DATA: 15 yr. olds
(Program for International Student Assessment)

A test of mathematical literacy for ***15*** year old students which focuses upon the direct application of mathematical principles. The test is administered every three years, with ***57*** countries participating in 2006. The test was ***not*** designed to measure curricular outcomes, but rather to assess mathematics within a real world context.

- **In 2006, the average U.S. score in mathematics literacy was 474, lower than the international average score of 498.**
- **Among the 57 countries in the sample, the U.S. was outperformed by 31 countries in math and 22 countries in science (m = 489).**
- **There was no measureable change in either the U.S. mathematics literacy score from 2003 to 2006, or the U.S. position compared to the international average.**

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PISA DATA: 15 yr. olds

Country	Average Score
International Average	498
Chinese-Taipai	549
Finland	548
Hong Kong-China	547
Korea	547
Netherlands	531
Switzerland	530
Canada	527
Macao-China	525
Liechtenstein	525
Japan	523
New Zealand	522
Belgium	520
Australia	520
Estonia	515
Denmark	513
Czech Republic	510
Iceland	506
Austria	505
Slovenia	504

PISA DATA: 15 yr. olds

Country	Average Score
International Average	498
Germany	504
Sweden	502
Ireland	501
France	496
United Kingdom	495
Poland	495
Slovak Republic	492
Hungary	491
Luxembourg	490
Norway	490
Lithuania	486
Latvia	486
Spain	480
Russian Federation	476
UNITED STATES	474

PISA Tragedy
National Science Board (2004)

- Enrollment in science and engineering programs is expanding ten times faster in China than in the United States.
- Approximately two-thirds of Chinese undergraduate students earn math, science, or engineering degrees compared to just one-third of United States undergraduate students.
- Science and engineering graduate programs have seen a 10 percent decline in the enrollment of U.S. students over the past decade, while enrollment of foreign graduate students has increased by 35 percent.
- Approximately 5.7 percent of 24 year-olds in the United States have a science or engineering degree, which is half that of Taiwan (11.1 percent), South Korea (10.9 percent), and the United Kingdom (10.7 percent). (Sivy, et al., 2004)
- Given the global demand for high tech workers, there is a greater exportation of jobs overseas due to a combination of cheaper wages, as well as a better educated workforce in mathematics and science.
- No Child Left Behind does not emphasize Science in the curriculum. Therefore, the applicational possibilities of mathematics often overlooked!

4 Reasons for U.S Decline

1. The language of math matters! Building number connections centered around a base-10 principle is crucial in the development of mathematical efficiency when problem solving.
2. Dry and boring material. Mathematical skill building needs to be FUN, and therefore needs to be presented in the format of games and activities.
3. Too much focus on the answers. In order to become facilitators of mathematical knowledge, students should practice multiple methods of problem solving from both a visual-spatial and verbal approach.
4. Time on task. Most elementary math instruction occurs in the afternoon, just 45 minutes per day.

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The Neural Machinery of Mathematics

Basic Terminology:

- ***Math Disability (Dyscalculia)***- **refers to children with markedly poor skills at deploying basic psychological processes used in computational problem solving (Haskell, 2000). These may include deficits with:**

  (1) Poor language and verbal retrieval skills
  (2) Working memory skills
  (3) Executive functioning skills
  (4) Faulty visual-spatial skills

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The Neural Machinery of Mathematics

Language Skills: **(temporal lobes)**

- **Most Asian languages have linguistic counting systems past *ten* (*ten-one, ten-two, etc)* whereas English deviates from base-10 system (Campbell & Xue, 2001).**
- **In English counting system, decades come first then unit *(i.e. twenty-one)* or sometimes this is reversed *(i.e. fifteen, sixteen, etc…)***
- **Chinese numbers are brief *(i.e. 4=si, 7=qi)* allowing for more efficient memory. By age four, Chinese students can count to 40, U.S. students to 15.**
- **U.S. kids spend 180 days in school**
  South Korea children spend 220 days in school
  Japan kids spends 243 days in school

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The Neural Machinery of Mathematics

Language Skills: (temporal lobes)

- Early math skills tend to be verbally encoded.
- Children with math disabilities frequently have delays in their language development. (Shalev et al, 2000)
- Word problems offer an intricate relationship between language and mathematics. Terms such as *all, some, neither, sum, etc.* may be confusing when embedded in the grammatical complexity of word problems (Levine & Reed, 1999).

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The Neural Machinery of Mathematics

Working Memory Skills: (Baddeley,1998)

- ***Phonological Loop*** - holds and manipulates acoustic information. Housed in *left temporal lobes*.
- ***Visual-Spatial Sketchpad*** - holds visual, spatial, and kinesthetic information in temporary storage by way of mental imagery. Housed along inferior portions of *right parietal lobes.*
- ***Central Executive System*** - command post for controlling two slave systems. Allocates attention resources whereby two cognitive tasks can be executed. Primarily housed in *frontal lobes*.
  - Central executive system serves to inhibit any negative distractors when problem solving (Hopko, 1998).

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Working Memory In The Brain

Working Memory System	Mathematical Skill
• *Phonological Loop*	• *Retrieval of math facts* • *Writing dictated numbers*
• *Visual-Spatial Sketchpad*	• *Mental math* • *Magnitude comparisons* • *Geometric Proofs*
• *Central Executive System*	• Inhibiting distracting thoughts • Modulating anxiety • Regulating emotional distress.

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The Neural Machinery of Mathematics

Executive Functioning Skills: (frontal lobes)

- Executive control mechanisms are a set of directive processes such as planning, self-monitoring, organizing, and allocating attention resources to effectively execute a goal directed task.
- Executive functioning dictates "*what to do when*", a critical process in solving word problems.
- Executive functioning allows students to choose an appropriate algorithm when problem solving.

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The Neural Machinery of Mathematics

EXECUTIVE DYSFUNCTION	BRAIN REGION	MATH SKILL
• *Selective Attention*	• *Anterior Cingulate/* • *Subcortical structures*	• Procedure/algorithm knowledge impaired • Poor attention to math operational signs • Place value mis-aligned
• *Planning Skills*	• *Dorsal-lateral PFC*	• Poor estimation • Selection of math process impaired • Difficulty determining salient information in word problems

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The Neural Machinery of Mathematics

EXECUTIVE DYSFUNCTION	BRAIN REGION	MATH SKILL
• *Organization Skills*	• *Dorsal-lateral PFC*	• Inconsistent lining up math equations • Frequent erasers • Difficulty setting up problems
• *Self-Monitoring*	• *Dorsal-lateral PFC*	• Limited double-checking of work • Unaware of plausibility to a response. • Inability to transcode operations such as (4X9) = (4X10) – 4

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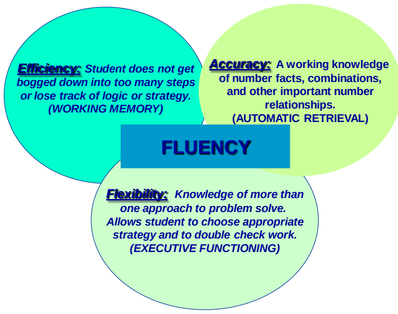
MATH FLUENCY (Russell, 1999)



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Three Basic Neural Codes to Format Numbers in the Brain

(1) ***Verbal Code*** - **numbers are encoded as sequences of words (*twenty-four* instead of 24). - Dehaene & Cohen, 1997**

- **Left temporal lobes.**
- **No need to understand quantitative concept.**
- **Main strategy used by younger children learning basic math facts (*two plus two equals four*)**
- **Math algorithms often preserved.**

Verbal Dyscalculia: **Students who have language based learning disabilities and struggle with retrieving symbols, albeit letters, words, or numbers.**

KEY CONSTRUCT: Language & Verbal Retrieval

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3 Subtypes of Math Disabilities

(1) *Verbal Dyscalculia Interventions:*
(Wright, Martland, & Stafford, 2000)

- **Distinguish between reciting *number words,* and *counting* (words correspond to number concept).**
- **Develop a FNWS and BNWS to *ten, twenty*, and *thirty* without counting back. Helps develop an automatic retrieval skills (Al's Game, Chris' Game, Chip's Game)**
- **Develop a base-ten counting strategy whereby the child can perform addition and subtraction tasks involving tens and ones.**
- **Reinforce the language of math by re-teaching quantitative words such as *more, less, equal, sum, altogether, difference, etc...* (April's Game)**

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3 Subtypes of Math Disabilities

(2) Procedural Code - numbers are encoded as fixed symbols representing a quantity of some sort, and sequenced in a particular order. (*24 instead of twenty-four*). - Von Aster, 2000

Procedural Dyscalculia Hierarchy:

- Preschool children need to take numbers from a *word level* to quantitative level by learning: *classification*, *ordering*, *one-to-one correspondence*, and *conservation*.
- Circuitry involves the syntactical arrangement of numerals. along our own internal number line.
- Critical in the execution of mathematical procedures for equations not committed to rote memory *(i.e. subtraction with regrouping, long division, etc…).*
- Bi-lateral occipital-temporal lobes store larger numbers!

Key Constructs: Working Memory and Anxiety

3 Subtypes of Math Disabilities

(2) *Procedural Dyscalculia Interventions*:

- Freedom from anxiety in class setting. Allow extra time for assignments and eliminate fluency drills.
- Color code math operational signs.
- Talk aloud all regrouping strategies.
- Use graph paper to line up equations.
- Adopt a curriculum such as *"Math Investigations"* which allows students to select their own algorithm.
- Attach number-line to desk and provide as many manipulatives as possible when problem solving.
- Teach skip-counting to learn multiplication facts.
- Teach patterns and relationships: (Melissa's Game, Mama's Game, Cordelia's Game, Habib's Game)

3 Subtypes of Math Disabilities

(3) *Magnitude Code* - numbers are encoded as analog quantities. Allows for value judgements, such as *"9"* is bigger than *"4"*. (Chocon, et al, 1999)

- Allows for semantic understanding of math concepts and procedures….."*Number Sense"*
- Allows for the evaluation of the plausibility of a response. (9 X 4 = 94)
- Allows for the transcoding of more challenging tasks into palatable forms of operations. For instance, 15 percent of 80 becomes 10 percent of 80 plus half the value.
- Bi-lateral inferior parietal lobes.

Key Constructs: IQ, Executive Functioning, Visual-Spatial

3 Subtypes of Math Disabilities

(3) ***Teaching "Number Sense"***:

- **Teach students to think in *"pictures"* as well as *"words"*.**
- **Have students explain their strategies when problem solving to expand problem solving options.**
- **Teach estimation skills to allow for effective previewing of response.**
- **Have students write a math sentence from a verbal sentence.**
- **Construct incorrect answers to equations and have students discriminate correct vs. incorrect responses.**
- **Incorporate money and measurement strategies to add relevance. Use *"baseball"* examples as well.**
- **(Heidi's Game, Dwain's Game)**

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Intervention Summary
(Horne & Feifer, 2007)

(1) Building number connections centered around a base-ten principle is crucial in the development of mathematical efficiency when problem solving.

(2) Mathematical skill building and developing a conceptual understanding of quantitative knowledge should be fun, self-motivating, and require far less effort when presented in the format of games and activities.

(3) In order to become facilitators of mathematical knowledge, students should practice multiple methods of problem solving by determining both a verbal and visual-spatial approach to solving addition, subtraction, multiplication, and division problems.

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National Mathematics Advisory Panel: Final Report (2008)

- **Streamline the curriculum with precise math "focal points" bringing each topic to closure before moving on.**
- **Fractions are skill lacking the most in U.S. students.**
- **Conceptual understanding, procedural fluency, and problem solving skills are mutually reinforcing. Educators should focus on a balanced curricular approach.**
- **Automaticity of facts frees up working memory for more complex problem solving.**
- **Teachers math knowledge is important for student achievement.**
- **Explicit instruction for students should be provided for struggling math students.**
- **Mathematically gifted students should be allowed to accelerate their learning.**
- **Teach in a hierarchy from non-symbolic to symbolic to patterns and relationships to conceptual understanding.**

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The 90 Minute Mathematics' Assessment

- Intelligence Tests
- Visual-Spatial Functioning
- Working Memory Capacity
- Attention
- Executive Functioning Skills
- Math Skills and Number Sense
- Math Anxiety Scale
- Developmental and School History

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Assessment Algorithm for Math

- PAL II:
  - Oral Counting
  - Fact Retrieval (Look & Write- Listen & Say)
  - Computational Operations
  - Place Value
  - Part-Whole Relationships
  - Finding the Bug
  - Multi-Step Problem Solving
  - Numeral Writing
  - Numeral Coding
  - Quantitative & Spatial Working Memory
  - Rapid Automatic Naming
  - Fingertip Writing

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